Efficient Stochastic FDTD Method for EM simulation

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Abstract **— Spectral polynomials expansion is incorporated in Finite-Difference Time-Domain method to perform uncertainty analysis in EM simulations. The generalized polynomial chaos (gPC) method is used as the expansion basis for parameter variations. Numerical examples are used to demonstrate the accuracy and efficiency of this proposed approach.**

I. Introduction

The finite difference time domain (FDTD) method has been successfully used to analyze very complex electromagnetic problems [1-2]. However, when it is used to analyze some complex physical systems whereuncertainty may exist in some design parameters, extensive electromagnetic simulations are often required. Traditionally, the most popular method for uncertainty analysis is Monte Carlo Method (MCM) in which the simulations are performed at the randomly sampled parameter spaces[3]. Although MCM is very straight forward, its relative slow converge rate makes it impossible to perform complex structure EM simulation to extract statistical information. Recently, there has been growing interest in the generalized polynomials chaos (gPC) method [4] for uncertainty modeling. Most of these efforts have been carried out in computational fluid dynamics (CFD) withsatisfactory results [5]-[6].

In this work, the gPC method, which is a spectral stochastic expansion approach, is applied in FDTD for stochastic computation of EM problems.

II. THOERY

The gPC method for computational analysis was first proposed by Xiu based on Wiener-Askey polynomials, which are a class of hypergeometic orthogonal polynomials.[4]. Each subset of Askey polynomials corresponds to a certain type of stochastic distribution and has a different set of optimal weighting function. As shown in [7], any second order stochastic process can converge in the $L_2(C)$ sense. Consider a random process $X(\theta)$, θ being the random event; it can be expressed using polynomial expansion as:

$$X(\theta) = \sum_{i=0}^{\infty} c_i \Phi(\boldsymbol{\xi}(\theta)) \tag{1}$$

Where $\boldsymbol{\xi}(\theta) = [\xi_1(\theta), \xi_2(\theta), \cdots \xi_N(\theta)]$ denotes a n-dimensional mutual independent random variable, which can be used to characterize the probability space [7].

The orthogonal property of polynomial basis function indicates that

$$\left\langle \Phi_i \Phi_j \right\rangle = \delta_{ij} \left\langle \Phi_i^2 \right\rangle \tag{2}$$

where the δ_{ij} is the Kronecker delta and the inner product is defined as:

$$\left\langle f(\boldsymbol{\xi}) g(\boldsymbol{\xi}) \right\rangle = \int_{\Omega} f(\boldsymbol{\xi}) g(\boldsymbol{\xi}) W(\boldsymbol{\xi}) d\boldsymbol{\xi} \tag{3}$$

Here Ω represents the event space and the weight function $W(\boldsymbol{\xi})$ can be properly chosen with respect to the basis polynomials $\{\Phi_i\}_{i=0}^{\infty}$.

Physical system often can be represented by a governing partial differential equation (PDE). The polynomial expansion can be projected into the PDE and get stochastic solution.

Consider a stochastic PDE has the forms of

$$L(u(x,t,\theta)) = f(x,t,\theta) \tag{4}$$

where $L(\cdot)$ is the differential operator; $f(x,t,\theta)$ is the excitation term; $u(x,t,\theta)$ is the stochastic solution of the PDE. Expanding the stochastic solution using corresponding Askey polynomial basis and using order of P to represent the expansion, we can have

$$u(x,t,\theta) = \sum_{i=0}^{P} \hat{u}_i(x,t) \Phi_i(\boldsymbol{\xi}(\theta)) \tag{5}$$

If the dimension of random variable $\boldsymbol{\xi}(\theta)$ is N and the highest order of polynomial to be used is M, then the total expansion order P is given by

$$P = \frac{(M+N)!}{M!N!} - 1 \tag{6}$$

Substituting (5) into (4) yields

$$L\left(\sum_{i=0}^{P} \hat{u}_i(x,t) \Phi_i(\boldsymbol{\xi}(\theta))\right) = f(x,t,\theta) \tag{7}$$

By taking the inner product on both sides of (7), we have

$$\left\langle L\left(\sum_{i=0}^{P} \hat{u}_i(x,t) \Phi_i(\boldsymbol{\xi}(\theta))\right), \Phi_k(\boldsymbol{\xi}(\theta)) \right\rangle = \left\langle f(x,t), \Phi_k(\boldsymbol{\xi}(\theta)) \right\rangle \tag{8}$$

k=0,1 , ……P

Due to the orthogonal property of polynomial basis, we can obtain P+1 equations for each coefficient $\hat{u}_i(x,t)$ .After all coefficients are obtained, statistic information can be extracted, such as mean and variance by:

$$E(u) = \hat{u}_o(x,t)$$
$$Var(u) = \sum_{i=1}^{P} \hat{u}_i^2(x,t) \tag{9}$$

By this spectral expansion approach, the original stochastic analysisis transformed to into obtaining the expansion coefficients. This can be efficiently implemented into the FDTD method where only one simulation is required. This is one of the advantages of this approach over other stochastic sampling method.

III. NUMERICAL RESULTS

To demonstrate the effectiveness of this proposed approach, the gPC-FDTD method is used to analyze the reflection and transmission coefficients of a dielectric slab as shown in figure 1. The height of the slab is 20 cm and the relative permittivity has Gaussian distribution with mean equal to 4 and standard variance equal to 0.4.The incident wave is derivative Gaussian pulse.

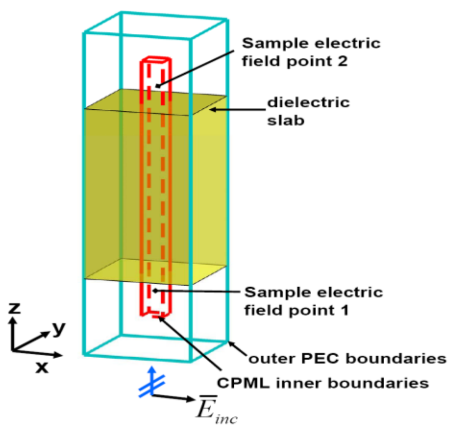

Fig. 1. Dielectric slab model

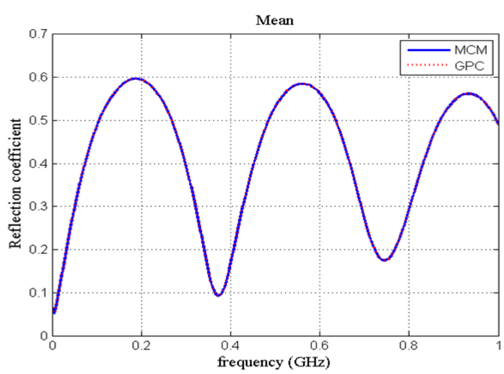

Fig. 2. Mean of reflection coefficient

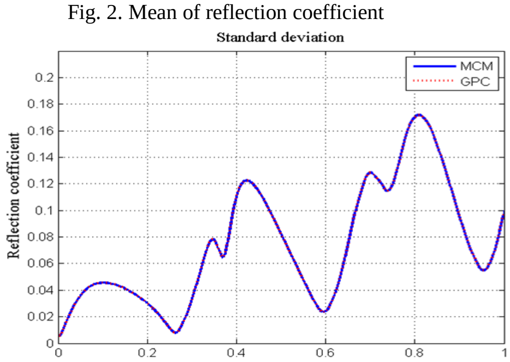

Fig. 3. Standard deviation of reflection coefficient

Figure 2 and Figure 3 show the mean and standard derivation of reflection coefficient with respect to frequency. Validations were performed by MCM with 400 realizations. As we can see from these figures, the gPC method has very good agreement of MCM. And table I shows the computational requirement of two different methods:

TABLE I

COMPUTATIONAL REQUIREMENT

Method	Number of Realizations	Total Time (h)	Memory (MB)
MCM	400	~ 13	~ 30
GPC	1	~ 0.95	~ 120

IV. CONCLUSION

The mathematical framework of gPC method is applied into FDTD for EM simulations. Numerical results indicate gPC method can accurately predict the stochastic solution for EM problems. Although more memory required, gPC proves an efficient way for stochastic EM computation in that it greatly reduces the simulation time. More details results and discuss will be included in the full version of this paper.

V. REFERENCES

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